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A ONE-DIMENSIONAL TRANSPORT CODE FOR FIELD-REVERSED CONFIGURATIONS*

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A radial transport code for Field-Reversed Configurations (FRCs) is described. Assuming quasi-neutrality and neglecting inertia and viscous force, the evolution of particle densities, temperatures, and magnetic field through a series of equilibrium states is simulated. To solve the equations, transformations of all dependent variables and the one independent variable are carried out. The processes of interest can be decoupled into two distinctive sets of equations that describe adiabatic and nonadiabatic processes. These sets of equations are then solved by two alternating steps: adiabatic and nonadiabatic.

Consider the FRC magnetic field geometry given in Fig. 1. Shown is a cylindrical (r,ϑ,z) coordinate system where we delineate regions I, II, and III by r_0, the radial location of the field null, r_s, the radial location of the separatrix, and r_w, the position of the wall. We make the approximation of straight field lines so that the magnetic field has one component that depends on r only. In addition we "slave" regions I and II by identifying radial positions with the same magnitude of magnetic flux measured from the field null as the same field line with the same density and temperature We use a flux-type independent variable where this identification becomes automatic. Although we neglect fieldline curvature, we allow for both changes in the axial length ℓ (this is necessary to satisfy axial force balance—the average beta condition) and separate axial energy and particle loss terms. This together with the requirement that our system of equations have no "explicit" ϑ or z dependence leads to magnetic and velocity fields $\underset{\sim}{B}$ and $\underset{\sim}{V}$ of the form $\underset{\sim}{B} = \left(0,0,B_z(r)\right)$ and $\underset{\sim}{V} = \left(V_r(r),0,V_z(r)z\right)$. With these approximations the Eulerian form of our model becomes:

$$\frac{\partial}{\partial t}\, rB_z + \frac{\partial}{\partial r}\, V_r\, rB_z = -\frac{\partial}{\partial r}\, r\, \frac{cR_\vartheta}{en} \qquad \text{(Ohm's + Ampere's laws)}, \tag{1}$$

$$p + \frac{B_z^2}{8\pi} = \text{const.} \quad \text{(Radial momentum balance)}, \tag{2}$$

$$\frac{\partial \zeta_i}{\partial t} + \frac{\partial}{\partial r} V_r \zeta_i = - \frac{V_z(z=\ell)}{\ell} \frac{\zeta_i}{r} + S_i \quad , \quad i = 1,2,3 \text{ (Transport)} \quad , \tag{3a-c}$$

where $\zeta_1 = rn$, $\zeta_2 = rT_e^{1/(\gamma-1)}$, $\zeta_3 = rT_i^{1/(\gamma-1)}$ (n is density; T_e and T_i are electron and ion temperatures, γ is the ratio of specific heats). The term cR_ϑ/en is the nonideal part of Ohm's law[1] and S_i contains all transport and radiation terms (e.g., thermal conduction, ohmic heating, electron-ion equilibration, axial loss, etc.—see Ref. 1). The first term on the right-hand-side of Eq. (3) is a source term due to axial length change (ζ_i is defined per unit lenght). The unknown $V_z(z = \ell)$ is determined by the average beta condition ($\langle\beta\rangle = 1 - r_s^2/2\, r_w^2$).[2]

Equations (1)-(3) describe evolution through a series of equilibrium states. Thus, Eq. (2) actually determines the radial velocity V_r. This rather nonnewtonian system of equations, which is the result of the neglect of inertia, comes from ignoring the fast timescale associated with the individual forces and, instead, following the long timescale associated with the time rate of change of these forces that are assumed to be in balance. That the forces in balance have time dependence induces a convective flow. This neglect of a fast timescale changes the usually hyperbolic mementum equation into the elliptic equation (instantaneous correlation of cause and effect) given as Eq. (2), and is a general feature of such an approximation.[3]

Equations (1)-(3) are most easily solved by a transformation to variables that in the absence of dissipation ($R_\vartheta = S_i = 0$) are adiabatic invariants. These invariants then evolve in time due to all forms of dissipation. This "flux variable" representation has been used previously.[4,5] We briefly outline new features relevant to the FRC.

In the flux variable procedure, we define $\psi = \psi(r,t)$ as $\psi = 2\pi\int_0^r r'B_z dr'/\psi_0$ ($\psi_0 = 2\pi\int_0^{r_0} rB_z dr$, so that ψ is normalized to have unit domain independent of flux decay at the field null $r = r_0$). Then, utilizing Eq. (1) to finish defining the appropriate partial derivatives (Eq. (1) is rewritten to define $V \equiv V_r + f(\zeta_i) \equiv \partial r/\partial t|_\psi$—the velocity of r at fixed ψ), we transform Eqs. (2)-(3) from the (r,t) to the $(\psi(r,t),t)$ representation. This transformation determines the new "adiabatic" dependent variables and eliminates V_r. These new variables are found to be $\xi_i \equiv \zeta_i \, dr/d\psi$. Thus,

$$\xi_i = \frac{\zeta_i dr}{2\pi r B_z dr/\psi_0} = \frac{\zeta_i \psi_0}{2\pi r B_z}$$

and utilizing $y = r^2/2$ in place of r (insures regularity at $r = 0$) we have $\partial y/\partial\psi = \psi_0/2\pi B_z$. ($\xi_1 = n\partial y/\partial\psi$, y is proportional to the volume of a flux surface). Thus ξ_1 and $\partial y/\partial\psi \to \infty$ as $B_z(r \to r_0) \to 0$ at the field null. Examination of the form of this singularity ($\xi_1 \to 1/(1-\psi)^{1/2}$ as $\psi \to 1$) suggests that we transform the variables ψ, ξ_i to the system $\eta \equiv -(1-\psi)^{1/2}$ $(-1 \leqslant \eta \leqslant 0)$, $\tilde{\xi}_i \equiv -\eta\xi_i$. Now the dependent variables $\tilde{\xi}_i$, $\partial y/\partial\eta$ and the transformed equations written in terms of these variables are everywhere nonsingular. The system of Eqs. (1)-(3) becomes

$$\frac{2^{\gamma}\tilde{\xi}_1 \left(\tilde{\xi}_2^{-\gamma-1} + \tilde{\xi}_3^{-\gamma-1}\right)}{(\partial y/\partial\eta)^{\gamma}} + \frac{\eta^2\psi_0^2}{8\pi^3 \, (\partial y/\partial\eta)^2} = \frac{\psi_{vac}^2}{32\pi^3(y_w - 2y_0)^2} \quad , \tag{4}$$

$$\frac{\partial\tilde{\xi}_i}{\partial t} - \frac{\partial}{\partial y}\frac{1}{2\eta}\left[\frac{(\eta^2-1)}{\psi_0}\frac{\partial\psi_0}{\partial t} - \frac{2\pi c y^{1/2} R_\vartheta}{en\psi_0}\right]\tilde{\xi}_i =$$

$$- \frac{V_z(z = \ell)}{\ell}\tilde{\xi}_i + \frac{1}{2} S_i \frac{\partial y}{\partial\eta} \quad , \qquad i = 1,2,3 \tag{5a-c}$$

In Eq. (4)(pressure balance), which has been written for a vacuum in region III, ψ_{vac} = vacuum flux, $y_w = r_w^2/2$ and $y_0 = r_0^2/2 = r_s^2/4$. Specifying $\tilde{\xi}_i$, ψ_0, ψ_{vac}, y_w, and a "guess" for y_0, Eq. (4), which is an algebraic equation for $\partial y/\partial\eta$, is solved for $\partial y/\partial\eta$. A solution is obtained when $\int_{\eta=-1}^{0} (\partial y/\partial\eta) \, d\eta$ equals the guessed value for y_0. The second term on the left-hand-side of Eqs. (5a-c) can be shown to be regular as $\eta \to 0$, and when regions I and II are "slaved" this flux vanishes at $\eta = 0$. To slave these regions, $y_{II} = 2y_0 - y_I$ is used, where $\tilde{\xi}_i$ are flux surface variables. Notice that in the ideal limit

$(R_\theta = S_i = \partial\psi_0/\partial t = 0)$ Eqs. (4)-(5) reduce to a simple form where no convective terms are present (Eq. (5) decouples from Eq. (4)).

After writing the explicit form of Eqs. (4)-(5) in an appropriate manner we separate the various terms in Eq. (5) into either convective, diffusive, or adiabatic in character. First, we advance a time step by solving Eqs. (4)-(5) keeping the convective and diffusive terms in Eq. (5). Secondly, we advance Eqs. (4)-(5) keeping only the adiabatic terms (sources and sinks) in Eq. (5). This completes one time step. In the first part Eqs. (4)-(5) are solved using a predictor-corrector method where Eq. (5) is implicitly differenced in block tridiagonal form.

Region III, the open field region, will not always be a vacuum region. In the latter case a separate flux variable is defined in this region and Eqs. (1)-(3) are appropriately transformed. (In this transformation it is important to use the fact that $\partial(\int_0^{r_s} rB_z dr)/\partial t = 0$.) We then require the continuity of the flux of all quantities at the separatrix.

References

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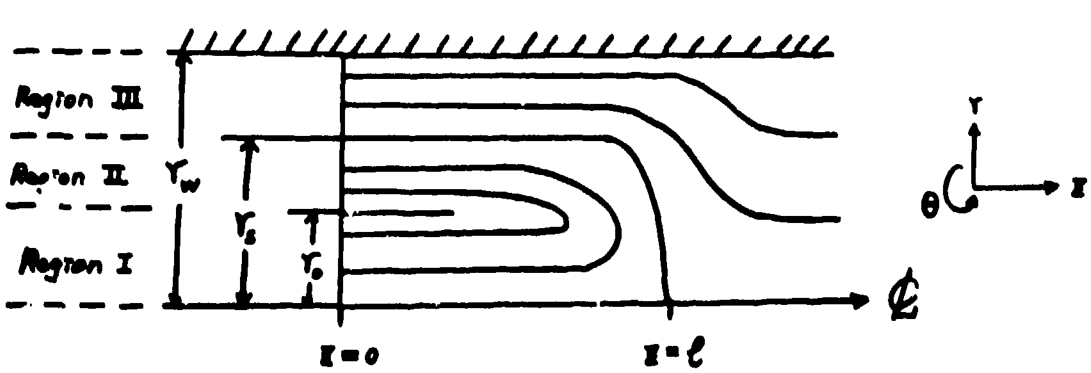

Fig. 1 FRC geometry.